School: Subject: Score:

26th Annual Greater Southeast Nebraska Mathematics Challenge
2019 Group Test

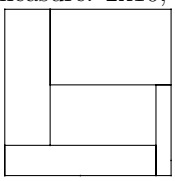
1. Starting in the top left corner of a 2x2 grid, and only being able to move to the right and down, there are exactly 6 routes to the bottom right corner. How many routes are there in a 3x3 grid? In a 4x4 grid?

   Note: A 1x1 grid shall be defined as:  And a 2x2 grid shall be defined as: 

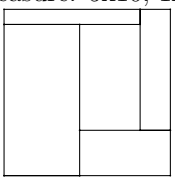
   There are 20 ways in a 3x3 and 70 ways in a 4x4. To get to the solution notice that combinations can be used. Regardless of the path that is selected in a 3x3 there will always be 3 "downs" and 3 "rights", thus 6 total paths will be used. By choosing the 3 "downs", the "rights" are then determined. The combination $\binom{6}{3}$ will equal the number of paths for a 3x3. Thus the number of paths for a 4x4 will be $\binom{8}{4}$. If one does not notice the use of the combinations the solution can also be found by using brute force drawing the possible paths and keeping count.

2. Find five rectangles whose dimensions are chosen from the integers $1, 2, 3, \ldots 9, 10$ (using each integer only once) and which can be put together to form a square. State the dimensions of each rectangle and draw a sketch of the square showing how the five rectangles are put together.

   First you must notice that the rectangles must be arranged in a spiralling pattern in order for them to form a square. Then you must also notice that the square must be 11x11. Then there are a few ways the problem can be done.
   The rectangles will measure: 2x10, 1x6, 5x8, 3x9, and 4x7.

    or

   The rectangles will measure: 5x10, 1x9, 3x6, 1x9, and 4x7.

   The square is a 11 x 11.

3. A group of 9 pirates stole a chest filled with gold. When they tried to split the gold evenly amongst themselves, they discovered there was one extra piece of gold. The pirates fought over the gold and ultimately one pirate was slain. When they again attempted to divide the stolen gold evenly amongst themselves, they again found that there was one extra piece of gold remaining. Another fight broke out and another pirate was slain. For a third time they attempted to split the gold evenly between themselves. This time they were successful. What is the smallest number of gold coins that the pirates could have stolen?

   This problem can be solved elegantly by using a system of modular equations. To solve using modular arithmetic use the following system. $x \equiv 1 \mod 9$, $x \equiv 1 \mod 8$, $x \equiv 0 \mod 7$ These are found by recognizing that $x = 9a + 1$
   $x = 8b + 1 \quad x = 7c$
   One can also solve this problem rather quickly by recognizing that if x must be one more than a multiple of 8 and one more than a multiple of 9 that it must be one more than a multiple of 72. Then list multiples of 72 and multiples of 7 until finding a multiple of 7 that is one more than the multiple of 72.
   The answer is 217 pieces of gold

4. A child has a piggy bank that automatically counts the dollar amount inside as well as displaying the number of coins. It currently shows there are 22 coins totaling to $2.28 inside of the piggy bank. There are 6 more dimes than quarters and 2 more nickels than pennies. How many of each type of coin does the piggy bank contain?

Let p=number of pennies, n=number of nickels, d=number of dimes, q=number of quarters. Now set up the proper set of equations. System of equations:

$$p + n + d + q = 22$$

$$1p + 5n + 10d + 25q = 228$$

$$d = q + 6$$

$$n = p + 2$$

Solve the system using any method of solving systems.
Solution:
4 quarters, 10 dimes, 5 nickels, and 3 pennies

5. Derrick and Miguel have always mowed the yard individually. Derrick can mow the yard in 4 hours and Miguel can mow the yard in 3 hours. If they were to work together how long would it take them to mow the yard? State your answer rounded to the nearest minute.

To solve this problem you must know the work equation: $\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t}$ where t_1 is the time it takes person 1 to complete a task, t_2 is the time it takes person 2 to complete a task, and t is the time it takes both people to complete a task when working together.

$$\frac{1}{4} + \frac{1}{3} = \frac{1}{t}$$

Solve the equation to get $t = \frac{12}{7}$ then change from hours into minutes 1 hour and 43 minutes or 103 minutes

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6. A cubical box has a ball that fits just inside of the box. The height, length, and width of the box are each equal to the diameter of the ball. What percent of the volume of the box is not occupied by the ball? (Use pi rounded to the nearest hundredth in the calculations).

Remember that $V_{box} = (2r)^3$ and that $V_{ball} = \frac{4}{3}\pi r^3$

$$\frac{V_{box}}{V_{ball}} = \frac{8r^3}{\frac{4}{3}\pi r^3} = \frac{\pi}{6}$$

Now you must realize that you have calculated the proportion of volume occupied by the ball. Therefore you need to subtract this value from 1.
Thus:
47.63% of the cube is not occupied by the ball.